

Refinements to the Probabilistic Approach to Fiscal Sustainability Analysis

Nathaniel Frank

Eduardo Ley

The World Bank
Economic Policy and Debt Department
September 2008



Abstract

This paper relaxes some key assumptions in the probabilistic approach to fiscal sustainability. First, the authors identify structural breaks over the sample period used to estimate the covariance matrix of the shocks to the debt ratios. Second, the assumption of normality

of the shocks is dropped by modeling their respective empirical distribution directly, which makes it possible to quantify asymmetries and thick tails. Third, the use of fiscal reaction functions is avoided by focusing attention on debt-stabilizing balances.

This paper—a product of the Economic Policy and Debt Department—is part of a larger effort in the department to study issues related to debt sustainability. Policy Research Working Papers are also posted on the Web at <http://econ.worldbank.org>. The author may be contacted at eley@worldbank.org.

The Policy Research Working Paper Series disseminates the findings of work in progress to encourage the exchange of ideas about development issues. An objective of the series is to get the findings out quickly, even if the presentations are less than fully polished. The papers carry the names of the authors and should be cited accordingly. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the International Bank for Reconstruction and Development/World Bank and its affiliated organizations, or those of the Executive Directors of the World Bank or the governments they represent.

Refinements to the Probabilistic Approach to Fiscal Sustainability Analysis

Nathaniel Frank

Nuffield College, University of Oxford, U.K.

Eduardo Ley

The World Bank, Washington DC, U.S.A.

Keywords. Fiscal sustainability, structural change, SVAR, bootstrap

JEL Classification System. F34, H60, C50

Address. nathaniel.frank@nuffield.ox.ac.uk and eley@worldbank.org

1. Introduction

The joint Bank-IMF debt-sustainability stress tests look at the debt-dynamics behavior under large changes of several of the variables involved, keeping all else constant. These shocks are applied to interest rates, exchange rates, or growth—without changing the values of other endogenous variables.

Celasun, Debrun and Ostry (2007, CDO henceforth) discuss the limitations of this approach and propose a probabilistic scenario analysis, using a stochastic framework to draw constellations of values for the variables driving the debt dynamics. For this purpose, CDO estimate a VAR for the non-fiscal components of the system. Under the assumption of joint normality they produce debt fan charts—by drawing from the fitted residual multivariate normal distribution, and iterating forward the law of motion of the debt-to-GDP ratio, along an estimated fiscal reaction function. Budina and van Wijnbergen (2007), and Bandeira *et al.* (2007) apply this framework to several country cases, and discuss how it improves the traditional Bank-IMF debt-sustainability framework.

This paper builds on this work, and explores some methodological refinements to the CDO fiscal sustainability analysis. First, we allow for structural breaks in the data-generation mechanism, which are identified through the application of Markov Switching models. Second, the assumption of normally distributed shocks is relaxed and bootstrapping techniques are used to make draws directly from their empirical distributions. This allows for a better risk assessment as the tail behaviour and asymmetries in the debt projections are more precisely quantified. Third, the estimation of the fiscal reaction function is replaced by inference with regard to the required primary balances consistent with sustainable fiscal policy.

This paper is organized as follows. Section 2 outlines the standard debt-sustainability framework and motivation of the research. Section 3 discusses the proposed methodology. Section 4 presents results for emerging market economies, and Section 5 conducts policy evaluation against a benchmark of fiscal sustainability. Finally, Section 6 concludes.

2. Debt-Sustainability Framework

Using d for debt and b for the government balance (expressed as percentages of GDP), g for real GDP growth, α^f for the share of foreign-denominated government debt, and β^f as the share of the tradable sector in GDP, the debt-dynamics are given by:

$$d_t = \frac{1 + i_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} - b_t \quad (1)$$

$$i_t = \hat{i}_t + e_t \alpha^f (1 + i_t^f) \quad (2)$$

$$\pi_t = \hat{\pi}_t + e_t \beta^f (1 + \pi_t^f) \quad (3)$$

where e_t is the rate of nominal depreciation of the local currency, and $\hat{i}_t = \alpha^h i_t^h + \alpha^f i_t^f$ and $\hat{\pi}_t = \beta^h \pi_t^h + \beta^f \pi_t^f$ are the effective interest rate and GDP deflator—which are weighted averages

We acknowledge useful comments from Fernando Blanco, Carlos Braga, Tihomir Stucka, Rogier van den Brink, Sweder van Wijnbergen, and participants at seminars at the World Bank and at the Banca d'Italia workshop on Fiscal Sustainability (Perugia, April 2008). We also thank Nina Budina, Oya Celasun and Xavier Debrun for useful conversations and sharing their data with us. The views expressed in this paper are the authors', and do not necessarily reflect those of the World Bank or its executive directors.

of domestic and foreign rates (Ley, 2006). Equations (1)–(3) can be combined to obtain the familiar expression driving public-debt dynamics:

$$d_t = \frac{1 + \hat{i}_t + e_t \alpha^f (1 + i_t^f)}{(1 + g_t)[1 + \hat{\pi}_t + e_t \beta^f (1 + \pi_t^f)]} d_{t-1} - b_t \quad (4)$$

The Bank-Fund joint debt-sustainability framework (DSF) analyses the behavior of equation (4) under some stress tests. Thus, large shocks are applied interest rates, exchange rates, or growth—keeping other endogenous variables fixed, and the subsequent behavior of the debt-dynamics is studied. CDO discuss the limitations of this approach and implement a probabilistic scenario analysis that addresses some of the shortcomings of the standard DSF.

2.1. Refinements

As mentioned, CDO develop a stochastic framework to draw joint shocks for the variables involved in (4). First, a VAR comprised of the aforementioned input variables such as growth, interest and depreciation rates is estimated. Second, a fiscal reaction function is computed, yielding the statistical relationship between the primary balances, past levels of debt and the output gap. Finally, shocks are repeatedly drawn from the approximated joint Normal distribution, based on the VAR-estimated covariance matrix, which are in turn used in the forward iteration of a debt equation and the fiscal reaction function. The resulting fan charts thus provide a probability distribution for future debt levels, which are contrasted against the DSF criteria.

In this paper methodological refinements are proposed, as we identify three issues which remain unaddressed by this technique.

- [1] **Structural Breaks.** Estimation conducted for emerging market economies often includes episodes that feature financial crises—*e.g.*, Mexico in 1995, Asia in 1997, Turkey in 2001 and Argentina in 2002. During such periods, significant jumps in interest rate spreads, exchange rates and GDP growth have historically been observed. This implies that the assumption of stationarity of the data generating processes cannot be ensured. In this context, estimation of a standard unvarying VAR is problematic. Consequently, inference with regard to the debt forecasts may be inappropriate, as the former is used in the input projections whereas the stochastic simulations are based on the latter. In this paper we allow for structural breaks which are identified through a Markov Switching VAR. Further estimation is then conducted over periods exhibiting constancy in the data generating processes, which ensures consistent coefficient and covariance estimates.
- [2] **Normality.** The assumption of normality with regard to the underlying shock distribution constitutes a severe limitation of the analysis in CDO. The resulting imposition of symmetry does not adequately reflect the empirical tail behaviour of economic variables, especially during period of financial distress. Furthermore, any inferred symmetry of the debt projections may merely be a construct of this assumption and thus conceals the true degree and shape of risk exposure. We replace the normality assumption by direct inference on the joint distribution of the shocks. Identification of stable sub-samples allows for the application of bootstrapping techniques. Consistent resampling of the data and repeated estimation implies that the resulting residuals converge to their true underlying counterpart. This then allows for the analysis of any

non-normality such as asymmetry and for the exact specification of the tail behaviour of the shocks.

- [3] **Fiscal Reaction Function.** CDO makes inference with regard to the dynamics between primary balances, lagged debt and the output gap by estimating a fiscal reaction function, which is subject to severe estimation and model uncertainty. In this paper we avoid its introduction by inverting equation (4). As a result, we specify a policy which is consistent with fiscal sustainability, such as debt stabilisation, and by drawing repeated shocks from the true error distribution, we make probabilistic inference with regard to the implied balances. This allows us to escape the aforementioned data constraints and for the evaluation of current policy against such a sustainability benchmark.

Other methodological differences introduced here include the amendment of the debt equation in order to account for tradables which are proxied by the export-to-GDP ratio. Furthermore, we do not assume that interest rate shocks affect the entire debt stock, but rather differentiate between its maturity structure such that merely the short term and a uniformly distributed fraction of the medium to long term debt is rolled over each period. Finally, with regard to data, our estimation is based on the fiscal panel and the input variables provided by CDO, except for the definition of the interest rate for foreign denominated debt. Whereas they employ the deflated US T-Bill rates as a proxy for borrowing costs in foreign markets, we incorporate interest rate spreads in the form of the Emerging Markets Bond Index (EMBI) as we believe that this more appropriate for emerging market countries.

3. Methodological Refinements

3.1. Markov Switching VAR

The Markov Switching VAR, as proposed by Hamilton (1989) and Krolzig (1997), differs from its linear counterpart by allowing the structural coefficients and the covariance matrix to be dependent on an unobserved state variable S_t , which is assumed to follow a k -dimensional Markov chain. This is of importance in our context, as we expect the joint distribution of the shocks to be non-constant across our sample period.

$$\begin{aligned} x_t &= \nu_{S_t} + \beta_{S_t}^1 x_{t-1} + \dots + \beta_{S_t}^q x_{t-q} + \varepsilon_t \\ S_t &\in \{S_1, S_2\} \\ \varepsilon_t &\sim (0, \Omega_{S_t}) \end{aligned} \tag{5}$$

Furthermore, estimation of this model allows for inference with regard to the probabilities of being in the respective states of the world, such that any structural breaks are identified by regime switches.

3.2. Bootstrap

As discussed above, this paper replaces the assumption of normally distributed shocks imposed in CDO by direct inference with regard to their true joint distribution. To this end we employ non-parametric bootstrapping techniques on the stationary sub-periods exhibiting constancy in terms

of the data generating processes. In this context, replication is conducted through a circular block bootstrap in order to account for dependence across time and individual variables, as suggested by Politis and Romano (1992). With regard to the selection of block length, a trade-off between the approximation of the observed data characteristics and the randomness of the resampling mechanism arises. Thus, its optimality is determined by an algorithm proposed by Politis and White (2004) whereby the length is based on the degree of autocorrelation exhibited by the data. We draw 1,000 pseudo series and sets of the residuals from their empirical distribution—which provides a consistent estimate of the true joint shock distribution.

4. Empirical Results

4.1. Smoothed Probabilities

In the estimation of the Markov Switching VAR, the existence of two data generating processes is assumed. Figure 1 presents evidence of structural breaks in the data, which suggests that estimation over the entire sample period potentially yields inconsistent parameter and covariance estimates, and thus incorrect debt projections.

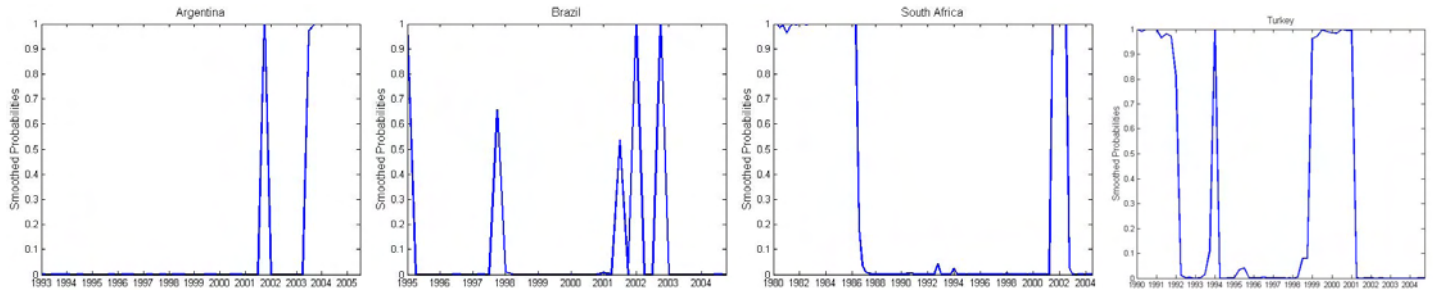


Figure 1. VAR regime-switching probabilities: Argentina, Brazil, South Africa and Turkey

- [1] In Argentina, a tranquil period between 1993 and 2002 is identified, after which statistically significant regime switches coincide with the observed period of financial instability. As a result, in what follows, we conduct estimation separately for these two distinct states of the world.
- [2] With regard to Brazil, significant spillovers are found during the Mexican and Argentinean Crises in 1995 and 2002, respectively, between which a sub-period of tranquility is defined, albeit a marginally significant break in 1997.
- [3] In South Africa, estimation is carried out across two stable periods, the former ranging from 1980 until 1987, and the latter from 1987 until 2001.
- [4] Finally, our data sample for Turkey exhibits structural breaks in 1992, 1994, 1998 and 2001, such that there is no evidence of a sustained period of stability. Thus we believe that the application of a stationary VAR is not appropriate—as constancy of the data generation mechanism cannot be ensured.

4.2. Argentina

As outlined above, estimation for Argentina is conducted separately for the two periods which correspond to times of tranquillity and financial distress, respectively. Figure 2 presents the resulting residuals from 1000 bootstrap replications. As indicated in the appendix, normality of the distributions is rejected jointly, which is also supported graphically in the presented quantile-quantile (QQ) plots in Figure 9, which plot the quantiles of each data set against the quantiles of a normal distribution with the same moments.

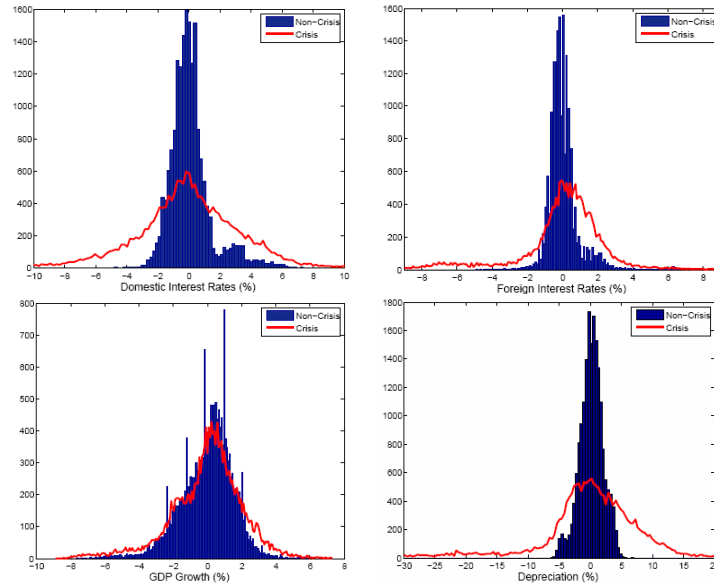


Fig. 2. Argentina—histograms of VAR residuals

Two further points of interest are to be noted. Firstly, during the non-crisis and the crisis periods, shocks are drawn from differing distributions, whereby the latter exhibits an up to five times greater variance relative to the former. This finding highlights the potential bias of estimating a VAR across the entire sample period, as in this case shocks would be drawn from a mixture of these two distributions, thus implying incorrect debt projections.

Secondly, evidence of asymmetric residuals is presented. During the tranquil times the interest rate distributions exhibit greater upside risk, whereas during the crisis period, the histogram for exchange rate movements is characterized by a long left hand tail. Clearly, the assumption of normality adopted in the existing literature does not allow for the quantification of this tail behaviour. Furthermore, this imposition of symmetry directly affects the probability distribution of the debt projections and thus leads to inappropriate conclusions with regard to fiscal sustainability.

The observed asymmetries and differing variances of the shock distributions are directly translated into the debt projections shown in Figure 3.¹ During the non-crisis period, the mean debt

¹ On the right panel, we represent boxplots for the debt distributions projected for different periods. Boxplots are useful tools to display empirical distributions without making any assumptions of the underlying statistical distribution. The lower side of the 'box' is the lower quartile (Q_1), while the upper side represents the upper quartile (Q_3); the red line dividing

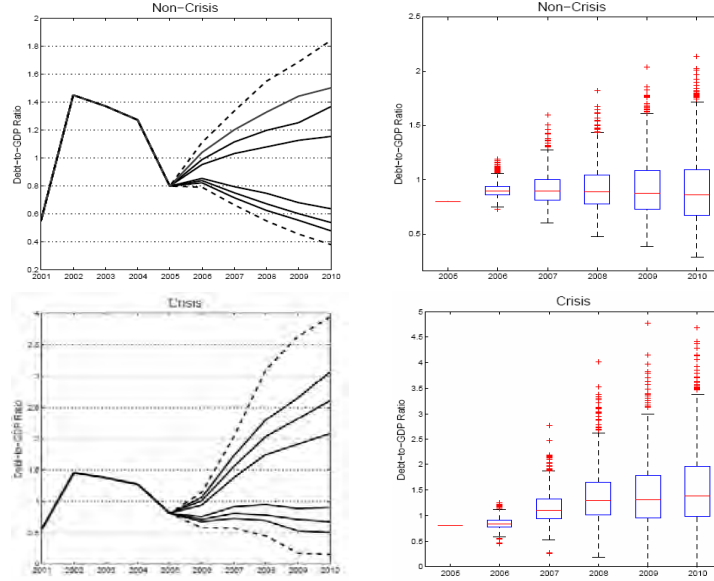


Fig. 3. Argentina—debt projections for tranquil and non-tranquil periods

level remains approximately constant, but there is evidence of upside risk in the form of the skewed probability distribution of future debt levels. This is driven primarily by the positive interest rate shocks illustrated in Figure 2. During the period of financial instability both the average debt level and the variance of the forecasts increase significantly, whereby the former is due to an upward jump in interest rate spreads raising the costs of borrowing. As a conclusion of this country example we argue that estimation over the entire sample period will, in addition to the aforementioned parameter inconsistency, incur an additional information loss. If the structural breaks remain unidentified, the resulting debt projections would incorrectly be comprised of a mixture of those obtained above.

4.3. Brazil and South Africa

Figure 4 displays replicated VAR residuals for Brazil. The most striking features with regard to asymmetries are the upside risk associated with interest rate changes, in addition to the long left hand tail in the distribution for exchange rates, indicating greater probability of a currency depreciation.

As in the case of Argentina, the assumption of joint normality is rejected. These asymmetries in the data are reflected in the debt projections in Figure 5. Whereas the mean level of future debt remains approximately constant, the box plots illustrate the significant probability mass corresponding to increased debt-to-GDP ratios.

As argued previously, for South Africa there appear to be two sub-samples exhibiting constancy in their data generating mechanisms. As a result, estimation is conducted for 1980–1987, which

the box represents the median (Q_2). The interquartile range (θ) is computed by subtracting the first quartile from the third quartile: $\theta = |Q_3 - Q_1|$. Any observation which lies outside the interval $[Q_1 - 1.5\theta, Q_3 + 1.5\theta]$ is considered an “outlier.” The smallest (and largest) observed value that is not an outlier is connected to the box with a horizontal line or “whisker.” Finally outlier observations are also represented as individual beyond the whiskers crosses.

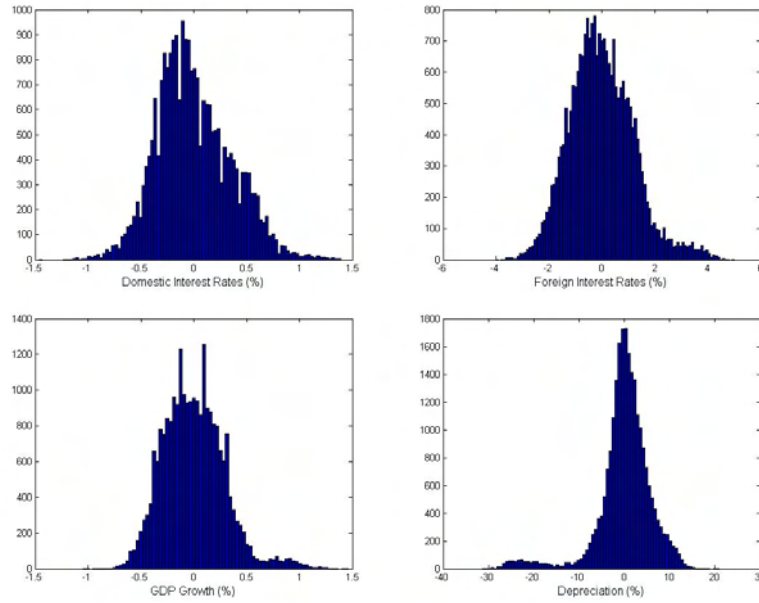


Figure 4. Brazil—histograms of VAR residuals

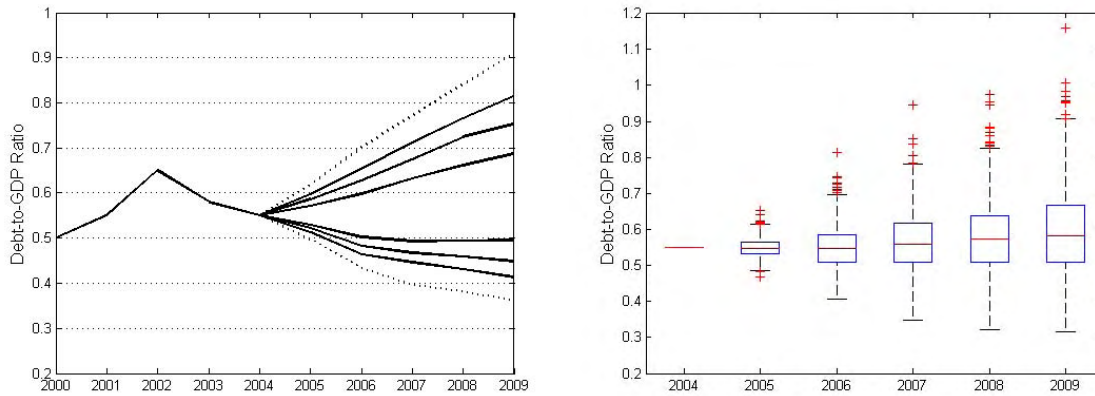


Figure 5. Brazil—debt projections

is defined as ‘period 1,’ and for 1987–2001; ‘period 2.’ This country case differs in terms of interpretation when compared to Argentina, where a distinct episode of financial distress is identified. From Figure 6 it follows that asymmetries in the VAR residuals are not as pronounced for South Africa, but that the variances differ across the two distributions. This corresponds to the debt projections, which exhibit differences in terms of the width of their respective confidence intervals. Also, it is noteworthy, that for both sub-periods the debt forecasts are downward sloping, a fact which is driven by low interest and high GDP growth rates.

5. Debt Stabilising Balances

The final departure from the technique proposed by CDO is that we avoid the use of fiscal reaction functions altogether. Their usage introduces unnecessary estimation and model uncertainty when attempting to make inference on the statistical relationship between primary balances, lagged debt

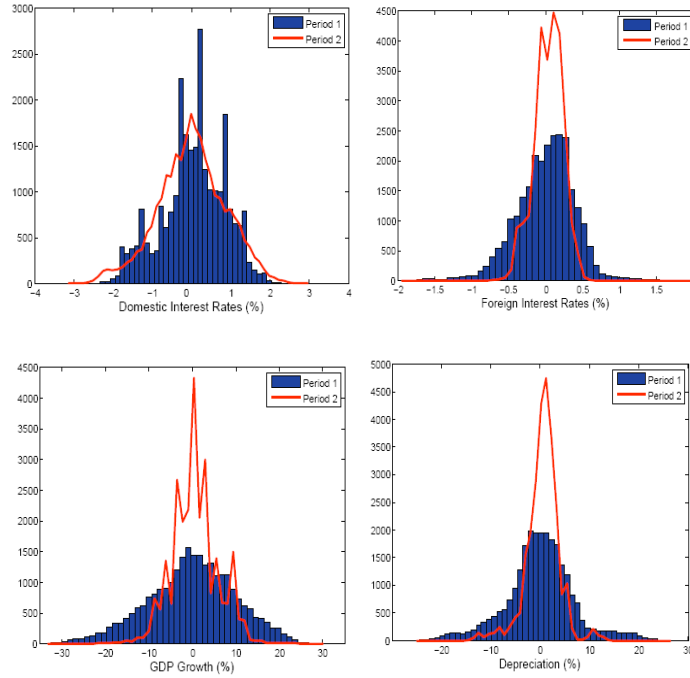


Fig. 6. South Africa—histograms of VAR residuals

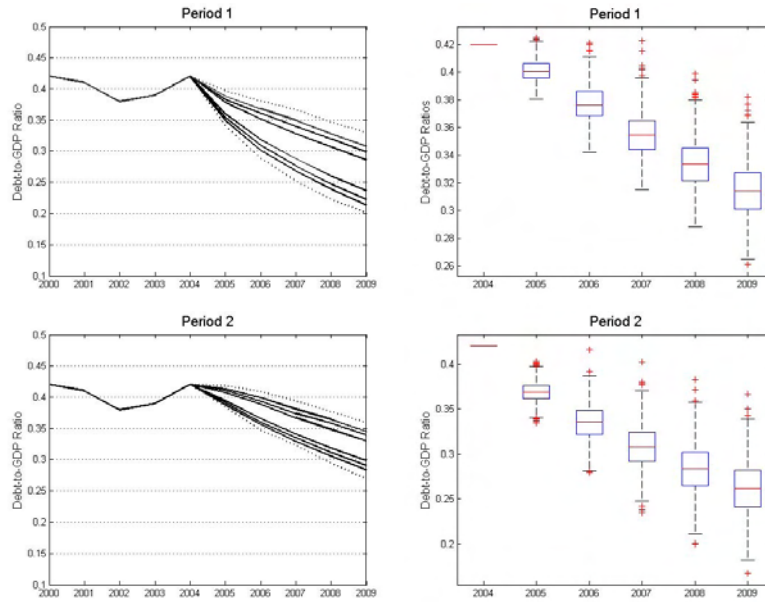


Figure 7. South Africa—debt projections

and output growth. In CDO, shocks are drawn from the joint distribution of the residuals, which in turn are utilized in the forward iteration of the fiscal reaction function and the debt equation.

Here we invert the problem by specifying policy which is consistent with fiscal sustainability,

such as debt stabilisation.² Rather than providing the probability distribution of future debt levels, we obtain the implied balances within this stochastic framework. We believe that this technique is of advantage, as in addition to overcoming data constraints affecting the fiscal variables, it allows for evaluation of currently implemented policies.

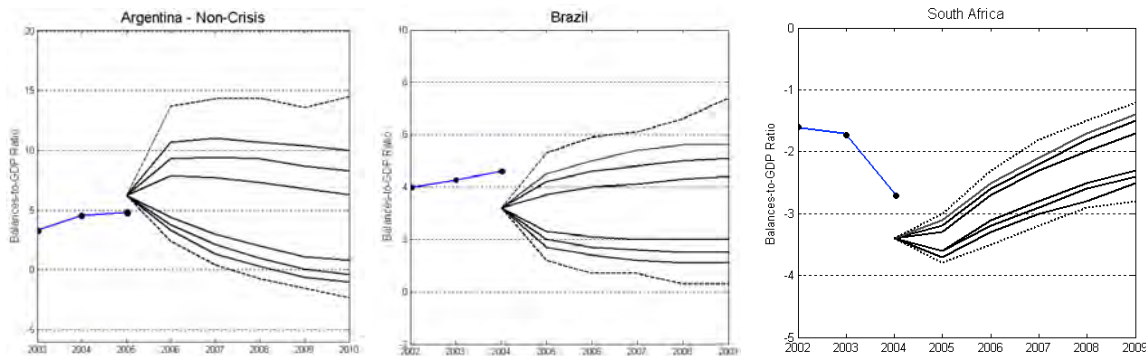


Figure 8. Debt-stabilization balances: Brazil, Argentina and South Africa

Figure 8 provides both the actual primary balances and those required in order to achieve debt sustainability under the true shock distribution. In Argentina and Brazil governments are currently achieving significant primary surpluses of over 4% of GDP, which is consistent with our notion of fiscal sustainability.

As pointed out above, the decreasing debt levels for South Africa are *mechanically* driven by the projected high levels of GDP growth and low interest rates. If these were indeed realistically expected to continue, the government could incur negative primary balances, whilst not endangering their future fiscal position. However, this is meant as an illustration of the limitations of a pure mechanical use of debt-sustainability framework.

6. Conclusion

This paper proposes methodological refinements to the fiscal sustainability analysis framework developed by Celasun, Debrun and Ostry (2007). Firstly, we pinpoint structural breaks in the data generating processes, which ensures consistency of parameter estimates. Furthermore, debt projections are obtained for distinct states of the world, rather than providing forecasts comprised of a mixture of these. Secondly, identification of stable sub-periods allows for the application of bootstrapping methods, such that the normality assumption with regard to shocks is replaced by inference on their true joint distribution. Finally, we avoid the estimation of a fiscal reaction function by inverting the debt equation. This allows us to overcome existing data constraints and provides a framework in which current policy can be evaluated against a fiscal sustainability measure.

² Given the complex form of the debt equation which accounts for differing maturity structures of debt, a simple inversion is not possible. Thus numerical techniques in the form of a grid search are employed.

7. Appendix — Normality Tests

The p -values for all joint normality test are smaller than 0.001. The QQ Normal plots below display the quantiles of each data set—tranquil and non-tranquil times residuals—against the quantiles of a normal distribution with the same moments. If the data were normally distributed, the graph would lie around the 45-degree reference line.

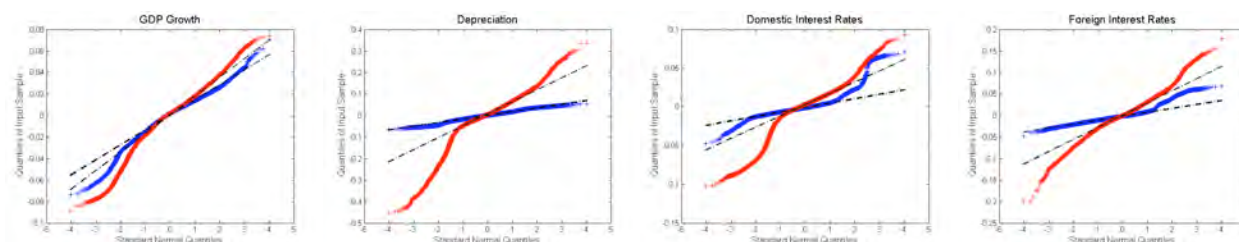


Figure 9. QQ Normal Plots of VAR Residuals—Argentina

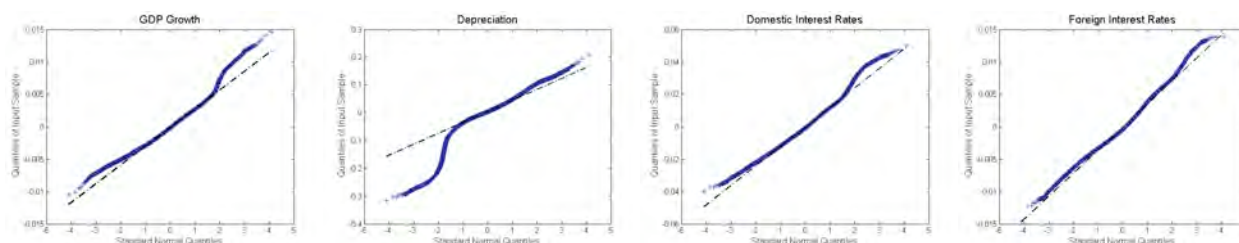


Figure 10. QQ Normal Plots of VAR Residuals—Brazil

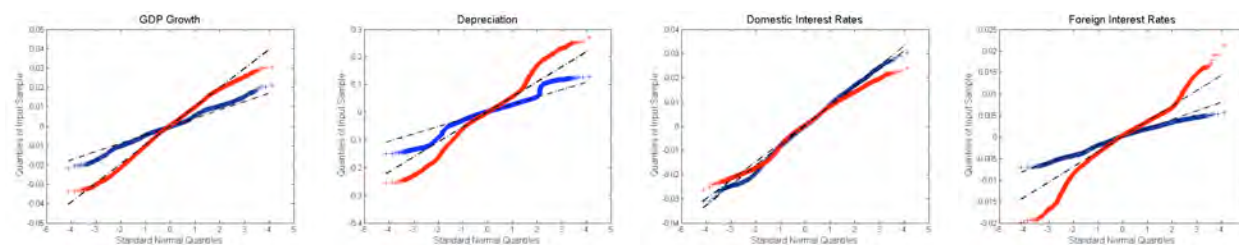


Figure 11. QQ Normal Plots of VAR Residuals—South Africa

8. References

- Bandiera, Luca, Nina Budina, Michel Klijn, and Sweder van Wijnbergen (2007) “The ‘How To’ of Fiscal Sustainability: A Technical Manual for Using the Fiscal Sustainability Tool” World Bank Policy Research Working Paper No 4170
- Budina, Nina, and Sweder van Wijnbergen (2007) “Quantitative Approaches to Fiscal Sustainability Analysis: A New WorldBank Tool Applied to Turkey” World Bank Policy Research Working Paper No 4169
- Celasun, Oya , Xavier Debrun, and Jonathan D. Ostry (2007) “Primary Surplus Behavior and Risks to Fiscal Sustainability in Emerging Market Countries: A Fan-Chart Approach,” *IMF Staff Papers*, 53(3): 401–425.
- Hamilton, J. D. (1989) “A new approach to the economic analysis of nonstationary time series and the business cycle,” *Econometrica* 57, 357–384.
- Krolzig, H.-M. (1997), *Markov Switching Vector Autoregressions: Modelling, Statistical Inference and Application to Business Cycle Analysis*, Lecture Notes in Economics and Mathematical Systems, 454, Springer-Verlag, Berlin.
- Ley, E. (2006) “Fiscal (and External) Sustainability,” (Unpublished, The World Bank), available at <http://129.3.20.41/eps/pe/papers/0310/0310007.pdf>
- Politis, D.N. and Romano, J. (1992) “A circular block-resampling procedure for stationary data,” in *Exploring the Limits of Bootstrap*, edited by Raoul LePage and Lynne Billard, John Wiley, 263–270.
- Politis, D. N. and H. White (2004) “Automatic block-length selection for the dependent bootstrap,” *Econometric Reviews* 23(1): 53–70. (See also: A. Patton, D.N. Politis, and H. White (2008) “Correction to: ‘Automatic Block-Length Selection for the Dependent Bootstrap’ by D. Politis and H. White.”)